Solving Equations

Kindergarten

NY-K.OA.6 Duplicate, extend, and create simple patterns using concrete objects.

First Grade

NY-1.OA.4 Understand subtraction as an unknown addend problem within 20. e.g., subtract $10 - 8$ by finding the number that makes 10 when added to 8.

NY-1.OA.7 Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. e.g., Which of the following equations are true and which are false? $6 = 6$ $7 = 8 - 1$ $5 + 2 = 2 + 5$ $4 + 1 = 5 + 2$

NY-1.OA.8 Determine the unknown whole number in an addition or subtraction equation with the unknown in all positions. e.g., Determine the unknown number that makes the equation true in each of the equations $8 + ? = 11$ $\_\_ - 3 = 5$ $6 + 6 = \square$

2nd Grade

NY-2.OA.3a Determine whether a group of objects (up to 20) has an odd or even number of members. e.g., by pairing objects or counting them by 2's.
NY-2.OA.3b Write an equation to express an even number as a sum of two equal addends.

NY-2.NBT.9 Explain why addition and subtraction strategies work, using place value and the properties of operations. Note: Explanations may be supported by drawings or objects.

3rd Grade

NY-3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities. e.g., using drawings and equations with a symbol for the unknown number to represent the problem.

NY-3.OA.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers. e.g., Determine the unknown number that makes the equation true in each of the equations $8 \times ? = 48$, $5 = \_\_ \div 3$, $6 \times 6 = ?$.

NY-3.OA.5 Apply properties of operations as strategies to multiply and divide. e.g.,
- If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication)
- $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication)
- Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property)

Note: Students need not use formal terms for these properties. Note: A variety of representations can be used when applying the properties of operations, which may or may not include parentheses.

NY-3.OA.6 Understand division as an unknown-factor problem. e.g., Find $32 \div 8$ by finding the number that makes 32 when multiplied by 8.

NY-3.OA.9 Identify and extend arithmetic patterns (including patterns in the addition table or multiplication table).

4th Grade

NY-4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, distinguishing multiplicative comparison from additive comparison. Use drawings and equations with a symbol for the unknown number to represent the problem.

NY-4.OA.5 Generate a number or shape pattern that follows a given rule. Identify and informally explain apparent features of the pattern that were not explicit in the rule itself. e.g., Given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.

5th Grade

NY-5.OA.1 Apply the order of operations to evaluate numerical expressions. e.g., • 6 + 8 ÷ 2 • (6 + 8) ÷ 2 Note: Exponents and nested grouping symbols are not included.

NY-5.OA.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. e.g., Express the calculation “add 8 and 7, then multiply by 2” as (8 + 7) × 2. Recognize that 3 × (18,932 + 921) is three times as large as 18,932 + 921, without having to calculate the indicated sum or product

NY-5.OA.3 Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. e.g., Given the rule “Add 3” and the starting number 0, and given the rule “Add 6” and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.

6th Grade

NY-6.EE.2 Write, read, and evaluate expressions in which letters stand for numbers.
NY-6.EE.2a Write expressions that record operations with numbers and with letters standing for numbers. e.g., Express the calculation “Subtract y from 5” as 5 – y
NY-6.EE.2b Identify parts of an expression using mathematical terms (term, coefficient, sum, difference, product, factor, and quotient); view one or more parts of an expression as a single entity. e.g., Describe the expression 2(8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.
NY-6.EE.2c Evaluate expressions given specific values for their variables. Include expressions that arise from formulas in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order (Order of Operations). e.g., Use the formulas V = s^3 and SA = $6s^2$ to find the volume and surface area of a cube with sides of length s = ½.
Note: Expressions may or may not include parentheses. Nested grouping symbols are not included.

NY-6.EE.5 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

NY-6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem. Understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

NY-6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form x + p = q; x – p = q; px = q; and x/□ = q for cases in which p, q and x are all nonnegative rational. Note: For the x/p = q case, p ≠ 0.

7th Grade

NY-7.EE.1 Add, subtract, factor, and expand linear expressions with rational coefficients by applying the properties of operations.

NY-7.EE.4a Solve word problems leading to equations of the form px + q = r and p(x + q) = r, where p, q, and r are rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. e.g., The perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width? Notes: The words leading to in the standard may require students to simplify or combine like terms on the same side of the equation before it is in the form stated in the standard. This standard is a fluency expectation for grade 7. For more guidance, see Fluency in the Glossary of Verbs Associated with the New York State Next Generation Mathematics Learning Standards.

NY-7.EE.4b Solve word problems leading to inequalities of the form px + q > r, px + q ≥ r, px + q ≤ r, or px + q < r, where p, q, and r are rational numbers. Graph the solution set of the inequality on the number line and interpret it in the context of the problem. e.g., As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions. Note: The words leading to in the standard may require students to simplify or combine like terms on the same side of the equation before it is in the form stated in the standard.

8th Grade

NY-8.EE.7 Solve linear equations in one variable.
NY-8.EE.7a Recognize when linear equations in one variable have one solution, infinitely many solutions, or no solutions. Give examples and show which of these possibilities is the case by successively transforming the given equation into simpler forms
NY-8.EE.7b Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and combining like terms. Note: This includes equations that contain variables on both sides of the equation.

NY-8.EE.8 Analyze and solve pairs of simultaneous linear equations.
NY-8.EE.8a Understand that solutions to a system of two linear equations in two variables correspond to points of

intersection of their graphs, because points of intersection satisfy both equations simultaneously. Recognize when the system has one solution, no solution, or infinitely many solutions.
NY-8.EE.8b Solve systems of two linear equations in two variables with integer coefficients: graphically, numerically using a table, and algebraically. Solve simple cases by inspection. e.g., 3x + y = 5 and 3x + y = 6 have no solution because 3x + y cannot simultaneously be 5 and 6. Notes: Solving systems algebraically will be limited to at least one equation containing at least one variable whose coefficient is 1. Algebraic solution methods include elimination and substitution. This standard is a fluency expectation for grade 8. For more guidance, see Fluency in the Glossary of Verbs Associated with the New York State Next Generation Mathematics Learning Standards.
NY-8.EE.8c Solve real-world and mathematical problems involving systems of two linear equations in two variables with integer coefficients. Note: Solving systems algebraically will be limited to at least one equation containing at least one variable whose coefficient is 1

NY-8.F.3 Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line. Recognize examples of functions that are linear and non-linear. e.g., The function A=s2 giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4), and (3,9), which are not on a straight line. Note: Function notation is not required in Grade 8

Algebra I: IF

AI-F.IF.7 Graph functions and show key features of the graph by hand and by using technology where appropriate. ★
(Shared standard with Algebra II)
AI-F.IF.7a Graph linear, quadratic, and exponential functions and show key features.
Notes:
Algebra I key features include the following: intercepts, zeros; intervals where the function is increasing, decreasing, positive, or negative; maxima, minima; and symmetries.
Exponential functions are of the form f(x) = a(b)x where a > 0 and b > 0 (b ≠ 1).
Graphing linear functions is a fluency recommendation for Algebra I. Students become fluent in solving characteristic problems involving the analytic geometry of lines, such as writing down the equation of a line given a point and a slope. Such fluency can support them in solving less routine mathematical problems involving linearity; as well as modeling linear phenomena.
AI-F.IF.7b Graph square root, and piecewise-defined functions, including step functions and absolute value functions and show key features.
Note: Algebra I key features include the following: intercepts, zeros; intervals where the function is increasing, decreasing, positive, or negative; maxima, minima; and symmetries.

AI-F.IF.8a For a quadratic function, use an algebraic process to find zeros, maxima, minima, and symmetry of the graph, and interpret these in terms of context. Note: Algebraic processes include but not limited to factoring, completing the square, use of the quadratic formula, and the use of the axis of symmetry.

AI-F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). (Shared standard with Algebra II) Note: Algebra I tasks are limited to the following functions: linear, quadratic, square root, piecewise defined (including step and absolute value), and exponential functions of the form f(x) = a(b)x where a > 0 and b > 0 (b ≠ 1).

Algebra I: Reasoning with Equations and Inequalities

AI-A.REI.1a Explain each step when solving a linear or quadratic equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

AI-A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. Note: Algebra I tasks do not involve solving compound inequalities.

AI-A.REI.4 Solve quadratic equations in one variable. Note: Solutions may include simplifying radicals
AI-A.REI.4a Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x-p)2 = q that has the same solutions. Understand that the quadratic formula is a derivative of this process.
Note: When utilizing the method of completing the square, the quadratic's leading coefficient will be 1 and the coefficient of the linear term will be limited to even (after the possible factoring out of a GCF). Students in Algebra I should be able to complete the square in which manipulating the given quadratic equation yields an integer value for q.
AI-A.REI.4b Solve quadratic equations by:
i) inspection,
ii) taking square roots,
iii) factoring,
iv) completing the square,
v) the quadratic formula, and
vi) graphing.
Recognize when the process yields no real solutions.
(Shared standard with Algebra II)
Notes:

Solutions may include simplifying radicals or writing solutions in simplest radical form.
An example for inspection would be x2 = 49, where a student should know that the solutions would include 7 and -7.
When utilizing the quadratic formula, there are no coefficient limits.
The discriminant is a sufficient way to recognize when the process yields no real solutions.

AI-A.REI.6a Solve systems of linear equations in two variables both algebraically and graphically. Note: Algebraic methods include both elimination and substitution.

AI-A.REI.7a Solve a system, with rational solutions, consisting of a linear equation and a quadratic equation (parabolas only) in two variables both algebraically and graphically. (Shared standard with Algebra II)

AI-A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane. Note: Graphing linear equations is a fluency recommendation for Algebra I. Students become fluent in solving characteristic problems involving the analytic geometry of lines, such as writing down the equation of a line given a point and a slope. Such fluency can support them in solving less routine mathematical problems involving linearity; as well as modeling linear phenomena.

AI-A.REI.11 Given the equations y = f(x) and y = g(x): i) recognize that each x-coordinate of the intersection(s) is the solution to the equation f(x) = g(x); ii)find the solutions approximately using technology to graph the functions or make tables of values; and iii) interpret the solution in context. (Shared standard with Algebra II)
Notes: Algebra I tasks are limited to cases where f(x) and g(x) are linear, polynomial, absolute value, and exponential functions of the form f(x) = a(b)x where a > 0 and b > 0 (b ≠ 1). Students should be taught to find the solutions approximately by using technology to graph the functions and by making tables of values. When solving any problem, students can choose either strategy.

AI-A.REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.
Note: Graphing linear equations is a fluency recommendation for Algebra I. Students become fluent in solving characteristic problems involving the analytic geometry of lines, such as writing down the equation of a line given a point and a slope. Such fluency can support them in solving less routine mathematical problems involving linearity; as well as modeling linear phenomena (including modeling using systems of linear inequalities in two variables).

Algebra II: Reasoning with Equations and Inequalities

AII-A.REI.1b Explain each step when solving rational or radical equations as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

AII-A.REI.2 Solve rational and radical equations in one variable, identify extraneous solutions, and explain how they arise. Note: Radical equations may include but are not limited to those of the form x3/5= 8 and 3x3/4+ 5 = 86.

AII-A.REI.4b Solve quadratic equations by: i) inspection, ii) taking square roots, iii) factoring, iv) completing the square, v) the quadratic formula, and vi) graphing. Write complex solutions in a + bi form. (Shared standard with Algebra I) Notes: • An example for inspection would be x2 = -81, where a student should know that the solutions would include ±9i. • An example where students need to factor out a leading coefficient while completing the square would be 4x2 + 8x - 9 = 0.

AII-A.REI.7b Solve a system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. (Shared standard with Algebra I) Note: Conics are limited to parabolas and circles.

AII-A.REI.11 Given the equations y = f(x) and y = g(x): i) recognize that each x-coordinate of the intersection(s) is the solution to the equation f(x) = g(x); ii) find the solutions approximately using technology to graph the functions or make tables of values; iii) find the solution of f(x) < g(x) or f(x) ≤ g(x) graphically; and iv) interpret the solution in context. ★ (Shared standard with Algebra I) Note: Tasks include cases where f(x) and/or g(x) are linear, polynomial, absolute value, square root, cube root, trigonometric, exponential, and logarithmic functions.

Algebra II: Linear, Quadratic and Exponential Models

AII-F.LE.2 Construct a linear or exponential function symbolically given: i) a graph; ii) a description of the relationship; and iii) two input-output pairs (include reading these from a table). (Shared standard with Algebra I)

AII-F.LE.4 Use logarithms to solve exponential equations, such as abct = d (where a, b, c, and d are real numbers and b > 0) and evaluate the logarithm using technology

AII-F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context. (Shared standard with Algebra I) Note: Algebra II tasks have a real-world context and exponential functions are not limited to integer domains.

Simplifying Expressions

6th Grade

NY-6.EE.3 Apply the properties of operations to generate equivalent expressions. e.g., Apply the distributive property to the expression 3(2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.

NY-6.EE.4 Identify when two expressions are equivalent. e.g., The expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y represents.

7th Grade

NY-7.EE.2 Understand that rewriting an expression in different forms in real-world and mathematical problems can reveal and explain how the quantities are related. e.g., a + 0.05a and 1.05a are equivalent expressions meaning that “increase by 5%” is the same as “multiply by 1.05.”

8th Grade

NY-8.EE.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions.
e.g., $3^2 \times 3^{-5} = 3^{-3} = \frac{1}{3^3} =$ 1/27.

Algebra I: SSE

AI-A.SSE.1 Interpret expressions that represent a quantity in terms of its context
AI-A.SSE.1a Write the standard form of a given polynomial and identify the terms, coefficients, degree, leading coefficient, and constant term
AI-A.SSE.1b Interpret expressions by viewing one or more of their parts as a single entity.
e.g., Interpret P(1 + r)^n as the product of P and a factor not depending on P.
Note: This standard is a fluency expectation for Algebra I. Fluency in transforming expressions and chunking (seeing parts of an expression as a single object) is essential in factoring, completing the square, and other mindful algebraic calculations.

AI-A.SSE.2 Recognize and use the structure of an expression to identify ways to rewrite it. (Shared standard with Algebra II)

e.g.,
$x^3 - x^2 - x = x(x^2 - x - 1)$
$53^2 - 47^2 = (53 + 47)(53 - 47)$
$16x^2 - 36 = (4x)^2 - (6)^2 = (4x + 6)(4x - 6) = 4(2x + 3)(2x - 3)$ or
$16x^2 - 36 = 4(4x^2 - 9) = 4(2x + 3)(2x - 3)$
$-2x^2 + 8x + 10 = -2(x^2 - 4x - 5) = -2(x - 5)(x + 1)$
$x^4 + 6x^2 - 7 = (x^2 + 7)(x^2 - 1) = (x^2 + 7)(x + 1)(x - 1)$

Note: Algebra I expressions are limited to numerical and polynomial expressions in one variable. Use factoring techniques such as factoring out a greatest common factor, factoring the difference of two perfect squares, factoring trinomials of the form ax2 +bx+c with a lead coefficient of 1, or a combination of methods to factor completely. Factoring will not involve factoring by grouping and factoring the sum and difference of cubes.

AI-A.SSE.3c Use the properties of exponents to rewrite exponential expressions. (Shared standard with Algebra II)

e.g.,
- $3^{2x} = (3^2)^x = 9^x$
- $3^{2x+3} = 3^{2x} \cdot 3^3 = 9^x \cdot 27$

Note: Exponential expressions will include those with integer exponents, as well as those whose exponents are linear expressions. Any linear term in those expressions will have an integer coefficient. Rational exponents are an expectation for Algebra II.

Algebra I: APR

AI-A.APR.1 Add, subtract, and multiply polynomials and recognize that the result of the operation is also a polynomial. This forms a system analogous to the integers. Note: This standard is a fluency recommendation for Algebra I. Fluency in adding, subtracting and multiplying polynomials supports students throughout their work in algebra, as well as in their symbolic work with functions.

AI-A.APR.3 Identify zeros of polynomial functions when suitable factorizations are available. (Shared standard with Algebra II) Note: Algebra I tasks will focus on identifying the zeros of quadratic and cubic polynomial functions. For tasks that involve finding the zeros of cubic polynomial functions, the linear and quadratic factors of the cubic polynomial function will be given (e.g., find the zeros ofP(x) = (x - 2)(x^2 - 9)).

Algebra I: IF

AI-F.IF.3 Recognize that a sequence is a function whose domain is a subset of the integers. (Shared standard with Algebra II)
<u>Notes:</u>

- Sequences (arithmetic and geometric) will be written explicitly and only in subscript notation.
- Work with geometric sequences may involve an exponential equation/formula of the form $a_n = ar^{n-1}$, where a is the first term and r is the common ratio.

Algebra I: Creating Equations

AI-A.CED.4 Rewrite formulas to highlight a quantity of interest, using the same reasoning as in solving equations. e.g., Rearrange Ohm's law V = IR to highlight resistance R.

Algebra II: SSE

AI-A.SSE.2 Recognize and use the structure of an expression to identify ways to rewrite it. (Shared standard with Algebra II)

e.g.,
$x^3 - x^2 - x = x(x^2 - x - 1)$
$53^2 - 47^2 = (53 + 47)(53 - 47)$
$16x^2 - 36 = (4x)^2 - (6)^2 = (4x + 6)(4x - 6) = 4(2x + 3)(2x - 3)$ or
$16x^2 - 36 = 4(4x^2 - 9) = 4(2x + 3)(2x - 3)$
$-2x^2 + 8x + 10 = -2(x^2 - 4x - 5) = -2(x - 5)(x + 1)$
$x^4 + 6x^2 - 7 = (x^2 + 7)(x^2 - 1) = (x^2 + 7)(x + 1)(x - 1)$

<u>Note:</u> Algebra I expressions are limited to numerical and polynomial expressions in one variable. Use factoring techniques such as factoring out a greatest common factor, factoring the difference of two perfect squares, factoring trinomials of the form ax2+bx+c with a lead coefficient of 1, or a combination of methods to factor completely. Factoring will not involve factoring by grouping and factoring the sum and difference of cubes.

AII-A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. (Shared standard with Algebra I)
AII-A.SSE.3a Factor a quadratic expression to reveal the zeros of the function it defines
AII-A.SSE.3c Use the properties of exponents to rewrite exponential expressions. (Shared standard with Algebra I) Note: Tasks include rewriting exponential expressions with rational coefficients in the exponent.

Algebra II: IF

AII-F.IF.7c Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
AII-F.IF.7e Graph cube root, exponential and logarithmic functions, showing intercepts and end behavior; and trigonometric functions, showing period, midline, and amplitude. Note: Trigonometric functions include sin(x), cos(x) and tan(x).

AII-F.IF.8 Write a function in different but equivalent forms to reveal and explain different properties of the function. (Shared standard with Algebra I)
AII-F.IF.8b Use the properties of exponents to interpret exponential functions, and classify them as representing exponential growth or decay. <u>Note:</u> Tasks also include real world problems that involve compounding growth/decay ($A = P(1 + (r/n))^{nt}$) and continuous compounding growth/decay (A=Pe^{rt}).

Algebra II: APR

AII-A.APR.2 Apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x).

AII-A.APR.3 Identify zeros of polynomial functions when suitable factorizations are available. (Shared standard with Algebra I)

AII-A.APR.6 Rewrite simple rational expressions in different forms; write (a(x))/(b(x)) in the form q(x) + $\frac{r(x)}{b(x)}$where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x).
<u>Note:</u> This standard is a fluency expectation for Algebra II. This standard sets an expectation that students will divide polynomials with remainders by inspection in simple cases. For example, one can view the rational expression $\frac{x+4}{x+3}$ as $\frac{(x+3)+1}{x+3}$which is $1 + \frac{1}{x+3}$.

Creating Functions

4th Grade
NY-4.MD.1 Know relative sizes of measurement units: ft., in.; km, m, cm e.g., An inch is about the distance from the tip of your thumb to your first knuckle. A foot is the length of two-dollar bills. A meter is about the height of a kitchen counter. A kilometer is 2 ½ laps around most tracks. Know the conversion factor and use it to convert measurements in a larger unit in terms of a smaller unit: ft., in.; km, m, cm; hr., min., sec. e.g., Know that 1 ft. is 12 times as long as 1 in. and express the length of a 4 ft. snake as 48 in. Given the conversion factor, convert all other measurements within a single system of measurement from a larger unit to a smaller unit. e.g., Given the conversion factors, convert kilograms to grams, pounds to ounces, or liters to milliliters. Record measurement equivalents in a two-column table. e.g., Generate a conversion table for feet and inches. NY-4.MD.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money. NY-4.MD.2a Solve problems involving fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. NY-4.MD.2b Represent measurement quantities using diagrams that feature a measurement scale, such as number lines. Note: Grade 4 expectations are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100. NY-4.OA.1 Interpret a multiplication equation as a comparison. Represent verbal statements of multiplicative comparisons as multiplication equations. e.g., - Interpret 35 = 5 x 7 as a statement that 35 is 5 times as many as 7 or 7 times as many as 5. - Represent “Four times as many as eight is thirty-two” as an equation, 4 x 8 = 32.
5th Grade
NY-5.MD.1 Convert among different-sized standard measurement units within a given measurement system when the conversion factor is given. Use these conversions in solving multi-step, real world problems. Notes: • All conversion factors will be given. • Grade 5 expectations for decimal operations are limited to work with decimals to hundredths. NY-5.NF.3 Interpret a fraction as division of the numerator by the denominator ($\frac{a}{b} = a \div b$). e.g., Interpret $\frac{3}{4}$ as the result of dividing 3 by 4, noting that $\frac{3}{4}$ multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size $\frac{3}{4}$. Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers. e.g., using visual fraction models or equations to represent the problem. e.g., If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie? NY-5.NF.5 Interpret multiplication as scaling (resizing). NY-5.NF.5a Compare the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. **e.g., In the case of 10 x $\frac{1}{2}$ = 5, 5 is half of 10 and 5 is 10 times larger than $\frac{1}{2}$.** NY-5.NF.5b Explain why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case). Explain why multiplying a given number by a fraction less than 1 results in a product smaller than the given number. Relate the principle of fraction equivalence $\frac{a}{b} = \frac{a}{b} \times \frac{n}{n}$ to the effect of multiplying $\frac{a}{b}$ by 1.**e.g., Explain why 4 × $\frac{3}{2}$ is greater than 4; Explain why 4 × $\frac{1}{2}$ is less than 4; $\frac{1}{3}$ is equivalent to $\frac{2}{6}$ because $\frac{1}{3} \times \frac{2}{2} = \frac{2}{6}$**
6th Grade
NY-6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. e.g., “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received three votes.” **NY-6.RP.2 Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0 (b not equal to zero), and use rate language in the context of a ratio relationship. e.g., “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there are ¾ cup of flour for each cup of sugar.” "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger." Note: Expectations for unit rates in this grade are limited to non-complex fractions.** **NY-6.RP.3a Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.** **NY-6.RP.3b Solve unit rate problems. e.g., If it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed? What is the unit rate? Note: Problems may include unit pricing and constant speed**

NY-6.RP.3c Find a percent of a quantity as a rate per 100. Solve problems that involve finding the whole given a part and the percent, and finding a part of a whole given the percent. e.g., 30% of a quantity means 30 100 times the quantity

NY-6.RP.3d Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities. Note: Conversion of units occur within a given measurement system, not across different measurement systems.

NY-6.EE.8 Write an inequality of the form $x > c$, $x \geq c$, $x \leq c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of these forms have infinitely many solutions; represent solutions of such inequalities on a number line.

NY-6.EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another. Given a verbal context and an equation, identify the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. e.g., In a problem involving motion at constant speed, list and graph ordered pairs of distances and times. e.g., Given the equation $d = 65t$ to represent the relationship between distance and time, identify t as the independent variable and d as the dependent variable.

7th Grade

NY-7.RP.1 Compute unit rates associated with ratios of fractions. e.g., If a person walks 1/2 mile in each ¼ hour, compute the rate as the complex fraction ½ / ¼ miles per hour, equivalently 2 miles per hour with 2 being the unit rate. Note: Problems may include ratios of lengths, areas, and other quantities measured in like or different units, including across measurement systems.

NY-7.RP.2 Recognize and represent proportional relationships between quantities
NY-7.RP.2a Decide whether two quantities are in a proportional relationship.
Note: Strategies include but are not limited to the following: testing for equivalent ratios in a table and/or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
NY-7.RP.2b Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
NY-7.RP.2c Represent a proportional relationship using an equation. e.g., If total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$.
NY-7.RP.2d Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.

NY-7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. Note: Examples of percent problems include: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, and percent error.

8th Grade:

NY-8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. e.g., Compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.

NY-8.EE.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y=mx$ for a line through the origin and the equation $y=mx+b$ for a line intercepting the vertical axis at b.

NY-8.F.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.
Notes: Function notation is not required in Grade 8.
The terms domain and range may be introduced at this level; however, these terms are formally introduced in Algebra I (AI-F.IF.1).

NY-8.F.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). e.g., Given a linear function represented by a table of values and a linear function represented by an algebraic equation, determine which function has the greater rate of change. Note: Function notation is not required in Grade 8.

NY-8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. Note: Function notation is not required in Grade 8.

NY-8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph. Sketch a graph that exhibits the qualitative features of a function that has been described in a real-world context. e.g., where the function is increasing or decreasing or when the function is linear or non-linear. Note: Function notation is not required in Grade 8.

Algebra I: Creating Equations
AI-A.CED.1 Create equations and inequalities in one variable to represent a real-world context. (Shared standard with Algebra II) Notes: - This is strictly the development of the model (equation/inequality). - Limit equations to linear, quadratic, and exponentials of the form f(x) = $a(b)^x$ where a > 0 and b > 0 (b ≠ 1). - Work with geometric sequences may involve an exponential equation/formula of the form $\boldsymbol{a_n = ar^{n-1}}$, where a is the first term and r is the common ratio. - Inequalities are limited to linear inequalities. - Algebra I tasks do not involve compound inequalities **AI-A.CED.2 Create equations and linear inequalities in two variables to represent a real-world context.** **Notes:** - **This is strictly the development of the model (equation/inequality).** - **Limit equations to linear, quadratic, and exponentials of the form f(x) = $a(b)^x$ where a > 0 and b > 0 (b ≠ 1).** **AI-A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. e.g., Represent inequalities describing nutritional and cost constraints on combinations of different foods.**
Algebra I: Interpreting Functions
AI-F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). Note: Domain and range can be expressed using inequalities, set builder notation, verbal description, and interval notations for functions of subsets of real numbers to the real numbers. **AI-F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context** AI-F.IF.3 Recognize that a sequence is a function whose domain is a subset of the integers. (Shared standard with Algebra II) Notes: ● Sequences (arithmetic and geometric) will be written explicitly and only in subscript notation. ● Work with geometric sequences may involve an exponential equation/formula of the form $a_n = ar^{n-1}$, where a is the first term and r is the common ratio. **AI-F.IF.4 For a function that models a relationship between two quantities:** **i) interpret key features of graphs and tables in terms of the quantities; and** **ii) sketch graphs showing key features given a verbal description of the relationship. (Shared standard with Algebra II)** **Notes:** - **Algebra I key features include the following: intercepts, zeros; intervals where the function is increasing, decreasing, positive, or negative; maxima, minima; and symmetries.** - **Tasks have a real-world context and are limited to the following functions: linear, quadratic, square root, piece-wise defined (including step and absolute value), and exponential functions of the form f(x) = $a(b)^x$ where a > 0 and b > 0 (b≠1).** AI-F.IF.5 Determine the domain of a function from its graph and, where applicable, identify the appropriate domain for a function in context. AI-F.IF.6 Calculate and interpret the average rate of change of a function over a specified interval. (Shared standard with Algebra II) Notes: - Functions may be presented by function notation, a table of values, or graphically. - Algebra I tasks have a real-world context and are limited to the following functions: linear, quadratic, square root, piecewise defined (including step and absolute value), and exponential functions of the form f(x) = $a(b)^x$ where a > 0 and b > 0, (b ≠ 1).
Algebra I: Building Functions
AI-F.BF.1 Write a function that describes a relationship between two quantities. (Shared standard with Algebra II) AI-F.BF.1a Determine a function from context. Define a sequence explicitly or steps for calculation from a context. (Shared standard with Algebra II) Notes: - Algebra I tasks are limited to linear, quadratic and exponential functions of the form f(x) = $a(b)^x$ where a > 0 and b > 0 (b ≠ 1). - Work with geometric sequences may involve an exponential equation/formula of the form an = $a_n = ar^{n-1}$, where a is the first term and r is the common ratio. - Sequences will be written explicitly and only in subscript notation.

AI-F.BF.3a Using f(x) + k, k f(x), and f(x + k):
i) identify the effect on the graph when replacing f(x) by f(x) + k, k f(x), and f(x + k) for specific values of k (both positive and negative);
ii) find the value of k given the graphs;
iii) write a new function using the value of k; and
iv) use technology to experiment with cases and explore the effects on the graph. (Shared standard with Algebra II)
Note: Tasks are limited to linear, quadratic, square root, and absolute value functions; and exponential functions of the form f(x) = a(b)x where a > 0 and b > 0 (b ≠ 1).

Algebra I: Linear, Quadratic, and Exponential Models

AI-F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.
AI-F.LE.1a Justify that a function is linear because it grows by equal differences over equal intervals, and that a function is exponential because it grows by equal factors over equal intervals.
AI-.F.LE.1b Recognize situations in which one quantity changes at a constant rate per unit interval relative to another, and therefore can be modeled linearly. e.g., A flower grows two inches per day.
AI-F-LE.1c Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

AI-F.LE.2 Construct a linear or exponential function symbolically given:
i) a graph;
ii) a description of the relationship;
iii) two input-output pairs (include reading these from a table). (Shared standard with Algebra II)
Note: Tasks are limited to constructing linear and exponential functions in simple context (not multi-step)

AI-F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

AI-F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context. (Shared standard with Algebra II) Note: Tasks have a real-world context. Exponential functions are limited to those with domains in the integers and are of the form f(x) = $a(b)^x$ where a > 0 and b > 0 (b ≠ 1).

Algebra II: Interpreting Functions

AII-F.IF.3 Recognize that a sequence is a function whose domain is a subset of the integers. (Shared standard with Algebra I)
Notes:

- In Algebra II, sequences will be defined/written recursively and explicitly in subscript notation.

This standard is a fluency expectation for Algebra II. Fluency in translating between recursive definitions and closed forms is helpful when dealing with many problems involving sequences and series, with applications ranging from fitting functions to tables to problems in finance

AII-F.IF.4 For a function that models a relationship between two quantities:
i) interpret key features of graphs and tables in terms of the quantities; and
ii) sketch graphs showing key features given a verbal description of the relationship. (Shared standard with Algebra I)
Notes:

- Algebra II key features include: intercepts, zeros; intervals where the function is increasing, decreasing, positive, or negative; relative maxima and minima; symmetries; end behavior; and periodicity.
- Tasks may involve real-world context and may include polynomial, square root, cube root, exponential, logarithmic, and trigonometric functions.

AII-F.IF.6 Calculate and interpret the average rate of change of a function over a specified interval. (Shared standard with Algebra I)
Notes:

- **Functions may be presented by function notation, a table of values, or graphically.**
- **Algebra II tasks have a real-world context and may involve polynomial, square root, cube root, exponential, logarithmic, and trigonometric functions.**

AII-F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). (Shared standard with Algebra I)
Note: Tasks may involve polynomial, square root, cube root, exponential, logarithmic, and trigonometric functions.

Algebra II: Creating Equations

AII-A.CED.1 Create equations and inequalities in one variable to represent a real-world context. (Shared standard with Algebra I)
Note: This is strictly the development of the model (equation/inequality). Tasks include linear, quadratic, rational, and exponential functions.

Algebra II: Building Functions
AII-F.BF.1 Write a function that describes a relationship between two quantities. (Shared standard with Algebra I) AII-F.BF.1a Determine a function from context. Determine an explicit expression, a recursive process, or steps for calculation from a context. (Shared standard with Algebra I) Notes: - Tasks may involve linear functions, quadratic functions, and exponential functions. - In Algebra II, sequences will be defined/written recursively and explicitly in subscript notation. AII-F.BF.1b Combine standard function types using arithmetic operations. e.g., Build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. AII-F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. Note: In Algebra II, sequences will be defined/written recursively and explicitly in subscript notation. AII-F.BF.3a Using f(x) + k, k f(x), f(kx), and f(x + k): i) identify the effect on the graph when replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); ii) find the value of k given the graphs; iii) write a new function using the value of k; and iv) use technology to experiment with cases and explore the effects on the graph. Include recognizing even and odd functions from their graphs. (Shared standard with Algebra I) Note: Algebra II tasks may involve polynomial, square root, cube root, exponential, logarithmic, and trigonometric functions. AII-F.BF.4a Find the inverse of a one-to-one function both algebraically and graphically AII-F.BF.5a Understand inverse relationships between exponents and logarithms algebraically and graphically. AII-F.BF.6 Represent and evaluate the sum of a finite arithmetic or finite geometric series, using summation (sigma) notation. AII-F.BF.7 Explore the derivation of the formulas for finite arithmetic and finite geometric series. Use the formulas to solve problems.